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A general CES-utility function and endogenous labour supply

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In the basic framework developed in the paper, we imposed perfect substitutability between consumption and leisure, exogenous labour supply and constant returns to scale in production. These assumptions are relaxed in this note in order to study the robustness of the previously derived results. For this aim, we allow for non-constant returns to scale in production by changing equation (5) in the main text into $X = hL_X^{\mu}$ (μ>0) and turn back to the CES-utility function - equation (1) in the paper - and consider the characteristics of the model for more general values of α. We first look at exogenous labour supply, and then we also consider endogenous labour supply.

Exogenous labour supply

By allowing for non-constant returns to scale in the production of X-goods, the mark-up pricing rule changes into (see Canton et al. 1999 for details)

$$p_X = \frac{\varepsilon}{\varepsilon - 1}\frac{w}{\mu h L_X^{\mu-1}}.$$

Along the lines set out in Appendix A.1 of the paper, the allocation of labour and consumption is then derived as

$$L_X = \frac{\theta\mu(\varepsilon-1)}{(1-\theta)\varepsilon - \theta\mu(\varepsilon-1)} \quad \text{and} \quad c = \bar{l}\left(\frac{\theta(\varepsilon-1)\mu h L_X^{\mu-1}}{\varepsilon N^{1-\sigma}}\right)^{\theta}(1-\theta)^{1-\theta}.$$

Solving the model for the critical productivity increase using the general utility function as specified in equation (1) in the paper and no longer imposing perfect substitutability between consumption and leisure activity yields the following F-function which solves for γ^*

$$\begin{aligned} F_{t_1,t_1-1} &= [(c^{A}_{t_1,t_1-1})^{\alpha} + \zeta(1-\bar{l}-\bar{s})^{\alpha}]^{1/\alpha} - [(c^{NA}_{t_1,t_1-1})^{\alpha} + \zeta(1-\bar{l})^{\alpha}]^{1/\alpha} \\ &+ \delta[(c^{A}_{t_1+1,t_1-1})^{\alpha} + \zeta(1-\bar{l})^{\alpha}]^{1/\alpha} - \delta[(c^{NA}_{t_1,t_1-1})^{\alpha} + \zeta(1-\bar{l})^{\alpha}]^{1/\alpha} = 0 . \end{aligned} \tag{2}$$

where $c^{A}_{t_1+1} = \gamma^{\theta} c^{NA}_{t_1+1} = \gamma^{\theta} c^{NA}_{t_1} = \gamma^{\theta} c^{A}_{t_1}$. Defining the consumption at t=t_1 as c, we can rewrite equation (1) as

$$\delta\left[\frac{\gamma^{\theta\alpha} c^{\alpha} + \zeta(1-\bar{l})^{\alpha}}{c^{\alpha} + \zeta(1-\bar{l})^{\alpha}}\right]^{1/\alpha} + \left[\frac{c^{\alpha} + \zeta(1-\bar{l}-\bar{s})^{\alpha}}{c^{\alpha} + \zeta(1-\bar{l})^{\alpha}}\right]^{1/\alpha} = 1 + \delta \ . \qquad (3)$$

This equation implicitly yields the solution for γ^*. Since the expression for γ^* cannot be solved in closed-form, we resort to a graphical and numerical analysis to describe the effects from intensified competition on the critical productivity increase at different degrees of substitution possibilities between consumption and leisure.

To understand the effects of competition on the willingness of consumers to adopt an improved technology at different degrees of substitution between leisure and consumption it is important to recognize that the decision to adopt affects consumers' utility in essentially two ways:

1. There is an instantaneous negative effect due to the loss in leisure associated with learning. This instantaneous effect is more negative if competition is more fierce as the reduction in leisure 'hurts' more if the consumption level is high.[1] The better substitutes leisure and consumption are, the less the instantaneous effect differs at different degrees of competition. In the limiting case of perfect substitutability considered above, the negative instantaneous effect is independent of competition. Hence, the negative learning effect is increasing in the degree of competition (unless α=1) as the reduction in leisure time associated with adoption is weighted more heavily.
2. There is a future positive effect of adoption on utility associated with increased productivity. This positive effect is stronger the larger the productivity increase and is reinforced by tougher competition.

There are thus two opposing effects of competition on the willingness to adopt. This is illustrated in Figures 1a-1c. These figures depict the *F*-function as a function of the productivity increase (γ) at different degrees of competition. Each figure represents a different case with respect to the elasticity of substitution between leisure and consumption (that is, $\alpha>0$, $\alpha=0$ and $\alpha<0$, respectively). The first effect just described is reflected in the fact that at a high degree of competition, *F* is relatively low at γ=1 (at which only the first, instantaneous effect is captured by *F*). If the productivity increase associated with adoption increases, *F* increases and the more so the stronger competitive forces are (reflected by the relative steepness of the two *F*-curves).

[1] Stated differently, the marginal utility of leisure positively depends on consumption, and consumption is higher in a more competitive environment.

Consumers are indifferent between adopting and keeping with the status quo if *F*=0. The answer to the question whether competition is good for growth depends on whether the *F*-curve associated with a high degree of competition intersects the dotted line, representing *F*=0, to the left of the intersection of the *F*-curve associated with the low degree of competition with the horizontal axis.

Figure 1a: *F*-function, α=0.5

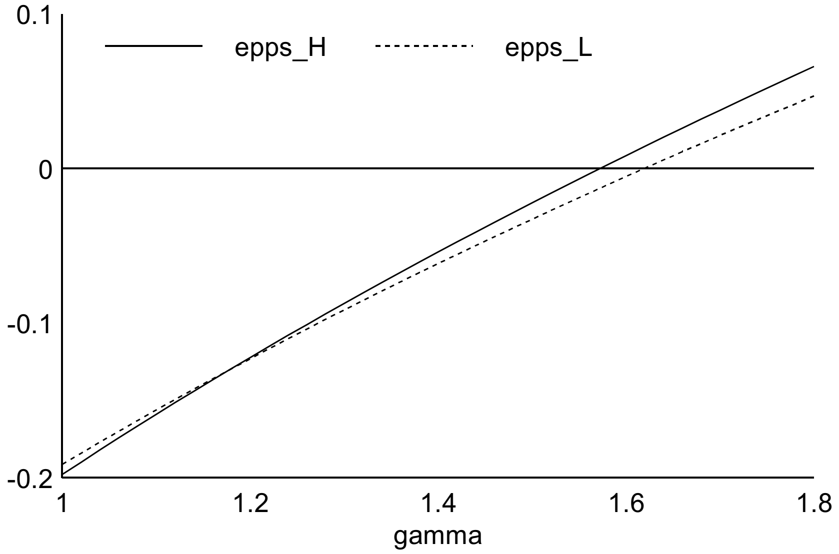


Figure 1b: *F*-function, α=0

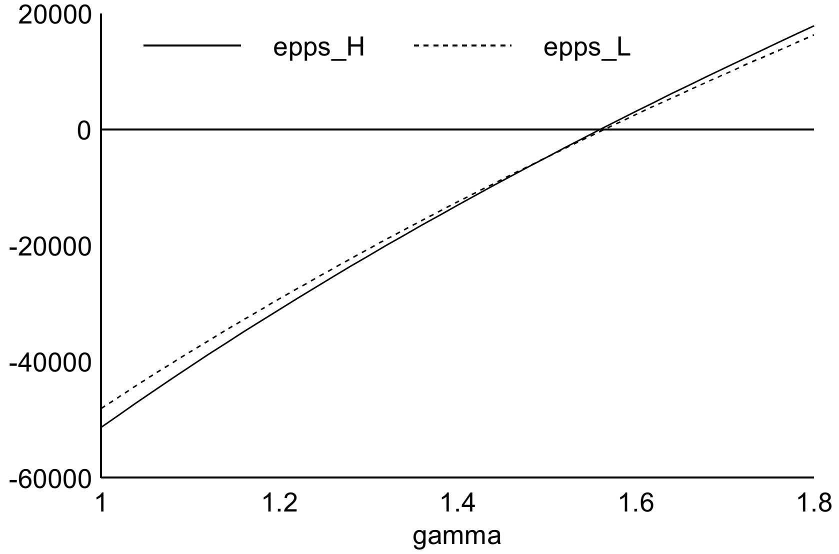
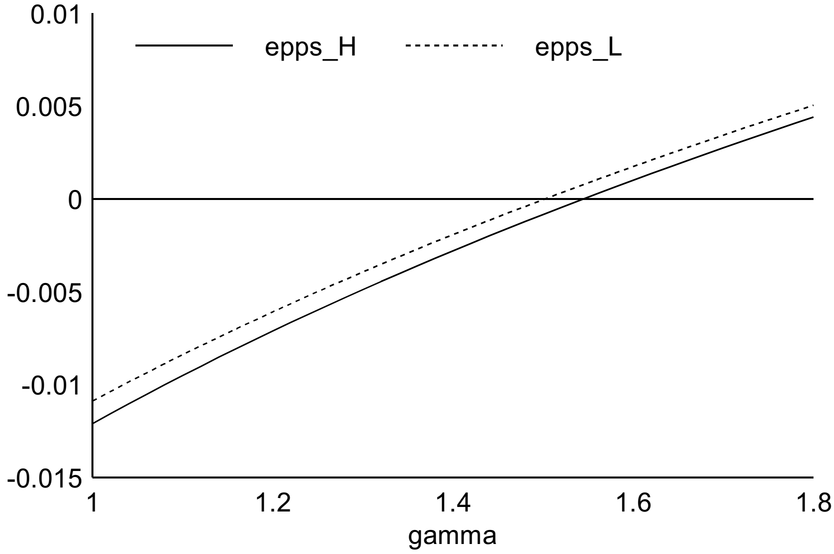


Figure 1c: *F*-function, α=-0.5



In general, we can show that competition has no effect on the willingness to adopt if α=0 (see below). In terms of our figures, this is depicted in Figure 1b in which the *F*-curves associated with different values of ε intersect the *F*=0 axis at one single point. The positive effect of competition on the willingness to adopt is, in other words, exactly offset by the negative (instantaneous) effect of competition on the willingness to adopt. In case leisure and consumption are relatively good substitutes (α>0) competition is good for growth (that is, the future positive effect of competition dominates). This is represented in Figure 1a. If on the other hand leisure and consumption are relatively bad substitutes (α<0) competition is bad for growth (that is, the instantaneous effect dominates) as depicted in Figure 1c.

These results are numerically illustrated in Table 1 which is based on the following parametrization of the model: $\bar{l}$ =0.5, $\bar{s}$ =0.1, μ=1.5, θ=0.5, *N*=1, ζ=1, δ=0.9, σ=1.3, *h*=1 and *L*=10. The entries in the Table give the solutions for the critical productivity increase for different combinations of the elasticity of substitution between leisure and consumption and the degree of competition.

Table 1. Critical productivity increase

	ε=3	ε=6
α=–0.5	1.50	1.54
α=0.02	1.56	1.56
α=0.5	1.62	1.56

An analytical solution can be obtained in the special (Cobb-Douglas) case in which $\alpha \to 0$. The utility function then simplifies to

$$u = c^{\beta} v^{1-\beta} \quad \text{where} \quad \beta \equiv \frac{1}{1+\zeta} \tag{3}$$

We can thus write the *F*-function as

$$F = c^{\beta}(1-\bar{l}-\bar{s})^{1-\beta} - c^{\beta}(1-\bar{l})^{1-\beta} + \delta[(\gamma^{\theta}c)^{\beta}(1-\bar{l})^{1-\beta} - c^{\beta}(1-\bar{l})^{1-\beta}] \tag{4}$$

where $c = c_{t_1}^{A} = c_{t_1}^{NA} = c_{t_1+1}^{NA} = c_{t_1+1}^{A}/\gamma^{\theta}$. Putting F equal to zero, the critical productivity increase is straightforwardly derived as

$$\gamma^{*} = \left[1 + \frac{1}{\delta}\left(1 - \left(\frac{1-\bar{l}-\bar{s}}{1-\bar{l}}\right)^{1-\beta}\right)\right]^{\frac{1}{\theta\beta}} > 1 \tag{5}$$

Note that this critical increase does not depend on the strength of competition as illustrated in Figure 1b.

Summarizing: competition is good for growth provided that consumers are sufficiently flexible in that they are willing to accept a current loss in leisure which is to be compensated by a future increase in consumption.

Endogenous labour supply

Next we consider the case of a CES-utility function and allow for endogenous labour supply. The aim of this is twofold. First, we intend to illustrate the robustness of previous results for the assumption of exogenous labour supply and second, we intend to illustrate that the results in the paper do not hinge on the assumption that the old bear adoption costs (provided that $\mu>1$).

The time constraint in this version of the model reads as

$$v = 1-l-s = \begin{cases} 1-l & \text{no schooling} \\ 1-l-\bar{s} & \text{schooling} \end{cases} \tag{7}$$

where the schooling cost only needs to be made by the young and the medium aged (those that will work with the new technology in the next period and thus have to change technology).

The individual consumer maximises his utility in four steps. In the first step he decides whether it is optimal to engage in resistance activity (we return to this step below). In the second step, optimal consumption and labour input are determined. The Lagrangian corresponding to this problem takes the following form

$$\mathcal{L} = \left[c(.)^{\alpha} + \zeta(1-l-s)^{\alpha}\right]^{1/\alpha} + \lambda\left(c(.)P_c - wl\right) . \tag{8}$$

The first order conditions are given by

$$\mathcal{L}_c:\ [c(.)^{\alpha} + \zeta(1-l-s)^{\alpha}]^{1/\alpha-1}c^{\alpha-1} + \lambda P_c = 0 , \tag{9}$$

$$\mathcal{L}_l:\ -[c(.)^{\alpha} + \zeta(1-l-s)^{\alpha}]^{1/\alpha-1}\zeta(1-l-s)^{\alpha-1} - \lambda w = 0 , \tag{10}$$

$$\mathcal{L}_{\lambda}:\ c(.)P_c - wl = 0 . \tag{11}$$

Elimination of the Lagrange multiplier λ from the first two FOCs yields

$$\left(\frac{c(.)}{1-l-s}\right)^{\alpha-1} = \zeta\frac{P_c}{w} , \tag{12}$$

or,

$$c(.) = (1-l-s)\left(\zeta P_c/w\right)^{1/(\alpha-1)} . \tag{13}$$

The third step of the optimization determines how much of consumption expenditures is spent on traditional and high-tech goods, respectively, while the fourth step determines how much of expenditures on high-tech goods is spent on varieties of the high-tech good. The solutions of these steps are as discussed in Appendix A.1 in the paper (but now for non-constant returns to scale in the production of *X*-goods; see also the case of exogenous labour supply discussed before). From the Cobb-Douglas structure of the composite good we have

$$P_c = \left(\frac{P_X}{\theta}\right)^{\theta}\left(\frac{P_Y}{1-\theta}\right)^{1-\theta} = w\left(\frac{N^{1-\sigma}\frac{\varepsilon}{\varepsilon-1}\frac{1}{\mu h L_{Xi}^{\mu-1}}}{\theta}\right)^{\theta}\left(\frac{1}{1-\theta}\right)^{1-\theta}. \tag{14}$$

Using the resulting expression for P_c/w we find

$$c(.) = (1-l-s)\left[\zeta\left(\frac{N^{1-\sigma}\varepsilon}{\theta(\varepsilon-1)\mu h L_{Xi}^{\mu-1}}\right)^{\theta}\left(\frac{1}{1-\theta}\right)^{1-\theta}\right]^{1/(\alpha-1)}. \tag{15}$$

Using income-spending equality $(P_c c = wl)$ we can derive (after some rearranging)

$$l = \frac{1-s}{1+\zeta^{1/(1-\alpha)}\left[\left(\frac{\theta(\varepsilon-1)\mu h L_{Xi}^{\mu-1}}{\varepsilon N^{1-\sigma}}\right)^{\theta}(1-\theta)^{1-\theta}\right]^{\alpha/(\alpha-1)}}, \tag{16}$$

where

$$L_{Xi} = \frac{\theta\mu(\varepsilon-1)L/N}{(1-\theta)\varepsilon+\theta\mu(\varepsilon-1)} \quad and \quad L = \sum_{\tau=0}^{2} P_{t_1-\tau} l^{A}_{t_1,t_1-\tau}. \tag{17}$$

In order to determine the equilibrium of the model with endogenous labour supply, it is crucial to recognize that there can exist two equilibria in the model:

(A) the medium-aged are in favour of adoption. In this case, the young will also be in favour of adoption and both engage in schooling after which adoption of the new technology takes place. The old disfavour adoption in any case as they have nothing to gain from it (under the assumption that $\mu>1$);

(NA) the medium-aged disfavour adoption. In this case, it is useless for the young to engage in schooling as the new technology will not be adopted in any case. So in this equilibrium, nobody engages in the costly activity of getting acquainted with the new technology.

These are the only two equilibria that can exist in the model. So what needs to be done is to determine the *F*-function for the medium-aged (taking into account that these are the only two equilibria) in order to determine whether adoption will take place or not. In the first equilibrium with adoption, we get

$$l^{A}_{t_1,t_1} = \frac{1-\bar{s}}{f^1(L_{Xit_1})},\ l^{A}_{t_1,t_1-1} = \frac{1-\bar{s}}{f^1(L_{Xit_1})},\ l^{A}_{t_1,t_1-2} = \frac{1}{f^1(L_{Xit_1})},\ L^{A}_{Xit_1} = f^2(L^{A}_{t_1}),\ L^{A}_{t_1} = \sum_{\tau=0}^{2} P_{t_1-\tau} l^{A}_{t_1,t_1-\tau}\ , \tag{18}$$

where

$$f^1(L_{Xit_1}) = 1 + \zeta^{1/(1-\alpha)}\left[\left(\frac{\theta(\varepsilon-1)\mu h L_{Xi}^{\mu-1}}{\varepsilon N^{1-\sigma}}\right)^{\theta}(1-\theta)^{1-\theta}\right]^{\alpha/(\alpha-1)} \quad \text{and} \quad f^2(L^{A}_{t_1}) = \frac{\theta\mu(\varepsilon-1)L^{A}_{t_1}/N}{(1-\theta)\varepsilon+\theta\mu(\varepsilon-1)}\ . \tag{19}$$

So we get a system of five equations in five unknowns ($l^{A}_{t_1,t_1},\ l^{A}_{t_1,t_1-1},\ l^{A}_{t_1,t_1-2},\ L^{A}_{Xit_1},\ L^{A}_{t_1}$) which can be solved and yields the solution for $c^{A}_{t_1,t_1-1}$. The solution for $c^{A}_{t_1+1,t_1-1}$ (with the new, higher level of productivity) is easily computed as before, under the assumption that no generation engages in schooling (using equations (14)-(16)). In the second equilibrium, with no adoption, we get

$$l^{NA}_{t_1,t_1} = \frac{1}{f^1(L^{NA}_{Xit_1})},\ l^{NA}_{t_1,t_1-1} = \frac{1}{f^1(L^{NA}_{Xit_1})},\ l^{NA}_{t_1,t_1-2} = \frac{1}{f^1(L^{NA}_{Xit_1})},\ L^{NA}_{Xit_1} = f^2(L^{NA}_{t_1}),\ L^{NA}_{t_1} = \sum_{\tau=0}^{2} P_{t_1-\tau} l^{NA}_{t_1,t_1-\tau}\ . \tag{20}$$

So we again get a system of five equations in five unknowns ($l^{NA}_{t_1,t_1},\ l^{NA}_{t_1,t_1-1},\ l^{NA}_{t_1,t_1-2},\ L^{NA}_{Xit_1},\ L^{NA}_{t_1}$) which can be solved and yields the solution for $c^{NA}_{t_1,t_1-1}$. The solution for $c^{NA}_{t_1+1,t_1-1}$ (with the old level of productivity) is again easily computed (using equations (14)-(16)). We can then establish the *F*-function as

$$\begin{aligned} F_t \ =\ & \delta[(c^{A}_{t+1,t-1})^{\alpha} + \zeta(1-l^{A}_{t+1,t-1})^{\alpha}]^{1/\alpha} + [(c^{A}_{t,t-1})^{\alpha} + \zeta(1-l^{A}_{t,t-1}-\bar{s})^{\alpha}]^{1/\alpha} - \\ & \delta[(c^{NA}_{t+1,t-1})^{\alpha} + \zeta(1-l^{NA}_{t+1,t-1})^{\alpha}]^{1/\alpha} - [(c^{NA}_{t,t-1})^{\alpha} + \zeta(1-l^{NA}_{t,t-1})^{\alpha}]^{1/\alpha} \ =\ 0\ . \end{aligned} \tag{21}$$

This expression can only be analysed numerically, but yields the solution for γ^*. The effects of competition on growth for different elasticities of substitution between consumption and leisure are again illustrated graphically and numerically as in the case of exogenous labour supply (with the same parameter values as used there, except of course for the labour supply which is now endogenous). Both from the graphs and the Table 2, it is evident that the results are qualitatively unaffected by the assumption of endogenous labour supply.

Figure 2a: *F*-function, α=0.5

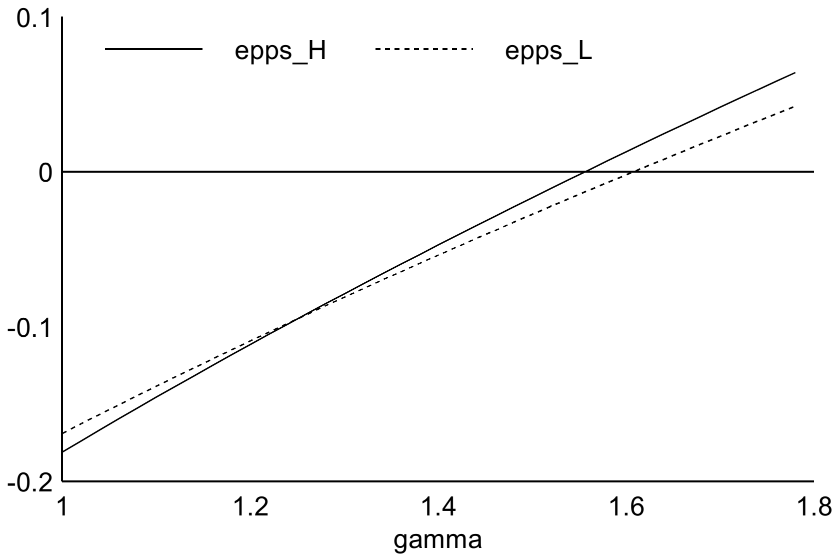


Figure 2b: *F*-function, α=0

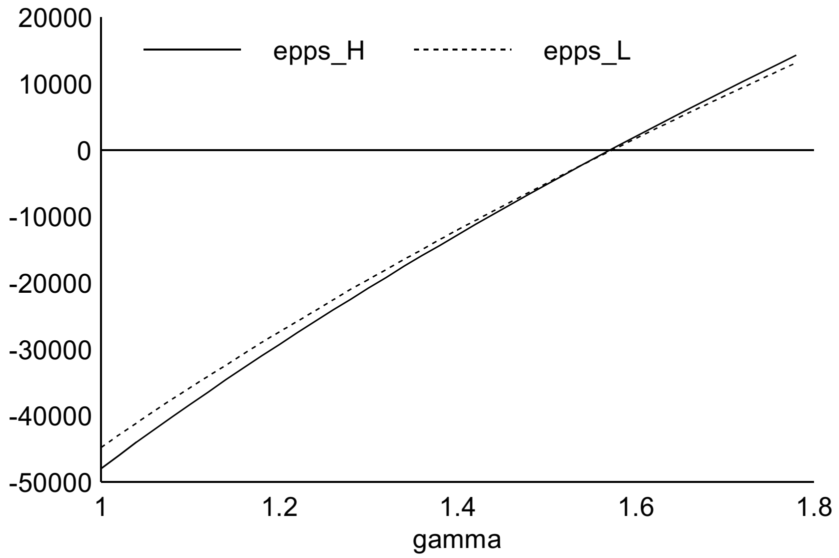


Figure 2c: *F*-function, α=-0.5

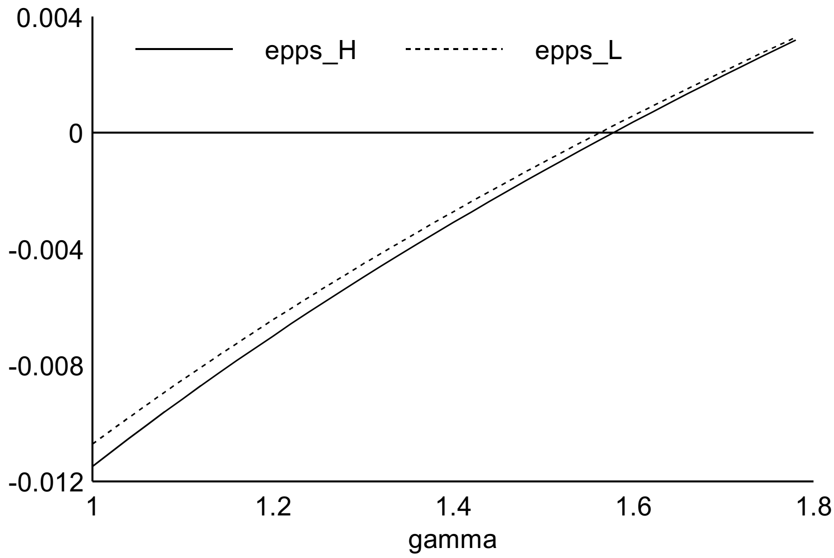


Table 2. Critical productivity increase

	ε=3	ε=6
α=–0.5	1.56	1.58
α=0.02	1.57	1.57
α=0.5	1.61	1.56

A second important lesson to be drawn from this exercise is that relaxing the assumption that the old generation has to engage in schooling although they have passed away when the technology becomes effective does not alter the results. The intuition is clear: despite the fact that the old do not lose time due to learning, they dislike the fact that the young and medium-aged learn as their labour supply reduces consequently (and hence the prices of the high-tech good increase). Note that for this result the non-linearity or scale effect introduced by the assumption that μ>1 is crucial.

References.

Canton, E.J.F., H.L.F. de Groot and R. Nahuis (1999): *Vested Interests and Resistance to Technology Adoption*, CentER Discussion Paper, no. 99106.